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Mathematical modelling of two-phase flow in a vertical well considering paraffin deposits and external heat exchange

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Abstract

Processes, occurring during exploration of gas-oil wells in frozen rock are simulated. A system of differential equations, describing hydro and thermal dynamics of an ascending two-phase flow of a hydrocarbon system in a vertical channel taking into account phase transitions and structure of a flow is developed. Kinetics of paraffin deposits on internal walls of an elevating column of a well are considered. The effect of the heat exchange of a well within frozen rock is developed using differential equations that describe the evolution of the radius of thermal influence of the well and the radius of the melting zone. We conclude with numerical research of some preventive ways of dealing with paraffin deposits.

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Keywords: Gas–liquid flow; Paraffin deposits; Well

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1. Introduction

Many petroleum deposits are characterised by the high content of heavy hydrocarbons (paraffin). The motion of such petroleum from a reservoir up to oil gathering systems is complicated by the formation of deposits consisting of heavy hydrocarbon in pipes. Results of work of the various researchers show that the paraffin deposits in a well are a collateral product of those complex natural and physical–chemical processes, which occur in petroleum as it rises to a surface. It is important to take into account melting zones and their influence on heat transfer. Substantiation of existing methods and the development of new methods of dealing with paraffin deposits in many respects depend on a detailed study of the process of paraffin loss in close interrelation with other processes, occurring in a well and around it. Such multilateral research is possible only within the framework of mathematical models, taking into account such interconnected circumstances as a current of gas–liquid flow in a well, heat exchange of a well with surrounding rock, and deposits of a solid phase (paraffin) on the internal walls of an elevating column.

2. A two-phase flow in an elevating column of a well

Processes occurring in active wells are so difficult and multiform, that one must first develop initial balance equations subject to various simplifications in each particular case. The final equations, which are used for the construction of appropriate mathematical models, should be defined by characteristics of the investigated process.

During the movement of a gas–liquid flow in a well the characteristic size of particles (bubbles of a gas, drops of a petroleum) is usually of substantially less diameter than the elevating column and other macroscopic scales of the considered flows. Therefore, as an elementary macrovolume for dealing with the appropriate

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equations, it is possible to choose a reasonably large volume including particles of both phases. One may describe a gas–liquid mix as a set of two continua, filling the same volume. At each point of volume, it is possible to enter the macroscopic velocities of phases, pressure, density of phases, and volumetric gas content, and to apply the usual methods of differential and integral calculations to the mathematical description of disperse systems.

It is also necessary to note that under constant external conditions, the non-stationary phenomena in an active well associated with start-up or transition to other modes of operation, are relatively short-term. Therefore, thermal and hydrodynamic modes of operation of a well can be considered almost stationary, and one can assume that the temperature in each section of a well is identical for each phase (gas and liquid).

The two-phase mix consists of three components: heavy, light, and some average component, both in a liquid phase, and as a kind of vapor in a gas phase. The heavy component can be present as a weighted solid phase and as dissolved in a liquid.

Let us direct the axis Z vertically upwards (Fig. 1), with its origin at the bottom of a well. Given the assumptions, and neglecting change of weight of a flow due to deposit of a solid phase, let us write an equation of conservation of mass in a form [7]

$$m_{\rm l} + m_{\rm g} = m = \text{const}, \tag{1}$$

$$k_{\rm l(g)} m_{\rm l} + k_{\rm g(g)} m_{\rm g} = m_{\rm (g)}, \tag{2}$$

where m_i $(i = {\rm l}, {\rm g})$ is the mass charge of the ith phase through section of a well with co-ordinate z; $k_{\rm l(g)}$ and $k_{\rm g(g)}$ are the mass concentrations of a light component in liquid and gas phases. The indices 'l' and 'g' pertain to liquid and gas phases. Thus, Eq. (1) expresses conservation of the flow of the entire mix, and (2) the conservation of the flow of a light component.

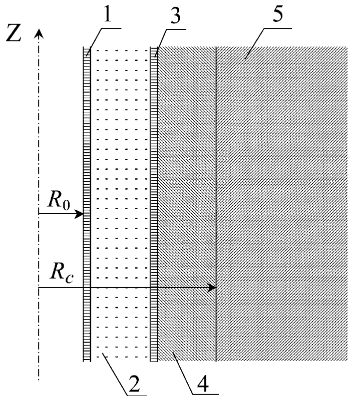

Fig. 1. A schematical section of a well: an elevating column (1), an annular space (2), a casing column (3), a cement ring (4), well environmental rock (5).

The equation of momentum for the entire mix in a stationary situation can be written as

$$\begin{aligned} & m_{\rm l}\frac{{\rm d}(v_{\rm l}^u)}{{\rm d}z} + m_{\rm g}\frac{{\rm d}(v_{\rm g}^u)}{{\rm d}z} \\ & \quad = -S\frac{{\rm d}p}{{\rm d}z} + J^{\rm lg}(v_{\rm l}^u - v_{\rm g}^u) - f_{\rm w} - J^{\rm ls}(v_{\rm ls} - v_{\rm l}^u) \\ & \qquad - (\rho_{\rm l}^0(1-\alpha) + \rho_{\rm g}^0\alpha)S_{\rm g}. \end{aligned} \tag{3}$$

Here ρ_i^0 $(i = {\rm l}, {\rm g})$ is the true density of phases; p the pressure; v_i^u the momentum-average velocity of the ith phase; S the area of section of a bore of a well; $f_{\rm w}$ the force of friction between a flow and by a wall of a channel; $J^{\rm lg}$ and $J^{\rm ls}$ the intensities of gas delivery and deposits, per unit of length of a channel; $v_{\rm ls}$ the velocity of heavy component concerning the liquid phase; α the volumetric gas content; g the acceleration due to gravity.

During operation of the well the velocity of a gas–liquid mix is usually considerably less than the velocity of sound, and consequently the item in the left-hand part of Eq. (3), connected with inertial effects, can be neglected. In addition, it is possible to neglect also "jet" forces $J^{\rm ls}(v_{\rm ls} - v_{\rm l}^u)$ and $J^{\rm lg}(v_{\rm l}^u - v_{\rm g}^u)$, connected with phase transitions. Then Eq. (3) will become [3,5]

$$\frac{{\rm d}p}{{\rm d}z} = -F_{\rm w} - \left(\rho_{\rm l}^0(1-\alpha) + \rho_{\rm g}^0\alpha\right)g \quad [F_{\rm w} = f_{\rm w}/S], \tag{4}$$

$$F_{\rm w} = \frac{\lambda_{\rm w}}{4R}\left(\rho_{\rm l}^0\frac{(1-\varphi)^2}{1-\alpha} + \rho_{\rm g}^0\frac{\varphi^2}{\alpha}\right)w^2 \quad [R = R_0 - \delta_S],$$

where φ is the flow-rate void fraction; $\lambda_{\rm w}$ the factor of friction between a flow and walls of a channel; w the outflow-average velocity; R_0 the radius of an elevating column; δ_S the thickness of paraffin deposits. Volumetric gas content differs from the flow-rate void fraction φ because of relative motion (slipping) of phases.

At cork and ring modes of flow for a factor of friction it is possible to use the expression

$$\lambda_{\rm w} = 0.067\left[\frac{158}{Re} + \frac{\varepsilon}{R}\right]^{0.2} \quad \left(Re = \frac{2\rho_{\rm l}^0(1-\alpha)wR}{\mu_{\rm l}}\right),$$

where $\mu_{\rm l}$ is the dynamic viscosity of a liquid, ε the size of a roughness of a pipe.

For dependence of a volumetric gas content we used a ratio offered by [1]

$$\alpha = \begin{cases} 0.833\varphi, & \varphi \leqslant 0.9, \\ \left[0.833\varphi + 0.167\left(1 + \frac{\rho_{\rm l}^0(1-\varphi)}{\rho_{\rm g}^0\varphi}\right)^{-1}\right]\varphi, & \varphi > 0.9, \end{cases}$$

$$\varphi = \left(1 + \frac{\rho_{\rm g}^0 m_{\rm l}}{\rho_{\rm l}^0 m_{\rm g}}\right)^{-1}.$$

The equation of heat inflow is

$$mc\frac{\mathrm{d}T}{\mathrm{d}z}=\frac{m_\mathrm{g}}{\rho_\mathrm{g}^0}\frac{\mathrm{d}p}{\mathrm{d}z}+m_\mathrm{l}\left(l^\mathrm{lg}_{(\mathrm{g})}-l^\mathrm{lg}_{(\mathrm{l})}\right)\frac{\mathrm{d}k_{\mathrm{l}(\mathrm{g})}}{\mathrm{d}z}+\left[l^\mathrm{lg}_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})}+l^\mathrm{lg}_{(\mathrm{l})}(1-k_{\mathrm{l}(\mathrm{g})})\right]\frac{\mathrm{d}m_\mathrm{l}}{\mathrm{d}z}-Q_\mathrm{w}, \quad (5)$$

$$mc=m_\mathrm{l}c_\mathrm{l}+m_\mathrm{g}c_\mathrm{g},\quad Q_\mathrm{w}=2\pi Rq_\mathrm{w}.$$

Here T is the average temperature of a two-phase flow; c_i $(i=\mathrm{l},\mathrm{g})$ are the specific heat of the ith phase; $l^\mathrm{lg}_{(\mathrm{l})}$ and $l^\mathrm{lg}_{(\mathrm{g})}$ the specific heat of evaporation of a liquid and allocation of a dissolved gas, respectively; q_w the heat flux, per unit length of a well.

From (1) and (2) it follows

$$\frac{\mathrm{d}m_\mathrm{l}}{\mathrm{d}z}=\left(m_\mathrm{l}\frac{\mathrm{d}k_{\mathrm{l}(\mathrm{g})}}{\mathrm{d}z}+m_\mathrm{g}\frac{\mathrm{d}k_{\mathrm{g}(\mathrm{g})}}{\mathrm{d}z}\right)\Big/\left(k_{\mathrm{g}(\mathrm{g})}-k_{\mathrm{l}(\mathrm{g})}\right). \quad (6)$$

Substituting (6) in (5), we obtain

$$mc\frac{\mathrm{d}T}{\mathrm{d}z}=\frac{m_\mathrm{g}}{\rho_\mathrm{g}^0}\frac{\mathrm{d}p}{\mathrm{d}z}+A_\mathrm{l}\frac{\mathrm{d}k_{\mathrm{l}(\mathrm{g})}}{\mathrm{d}z}+A_\mathrm{g}\frac{\mathrm{d}k_{\mathrm{g}(\mathrm{g})}}{\mathrm{d}z}-Q_\mathrm{w}, \quad (7)$$

$$A_\mathrm{l}=m_\mathrm{l}\left(l^\mathrm{lg}_{(\mathrm{g})}-l^\mathrm{lg}_{(\mathrm{l})}\right)+\frac{l^\mathrm{lg}m_\mathrm{l}}{k_{\mathrm{g}(\mathrm{g})}-k_{\mathrm{l}(\mathrm{g})}},\quad A_\mathrm{g}=\frac{l^\mathrm{lg}m_\mathrm{l}}{k_{\mathrm{g}(\mathrm{g})}-k_{\mathrm{l}(\mathrm{g})}},$$

$$l^\mathrm{lg}=l^\mathrm{lg}_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})}+l^\mathrm{lg}_{(\mathrm{l})}\left(1-k_{\mathrm{l}(\mathrm{g})}\right).$$

For dependence of the partial pressure of a light component on its mass concentration in a liquid phase we shall accept the Henry law

$$p_{(\mathrm{g})}=G_{(\mathrm{g})}k_{\mathrm{l}(\mathrm{g})}, \quad (8)$$

where $G_{(\mathrm{g})}$ is the constant of Henry.

We shall set the partial pressure of vapor (average component) equal the pressure of saturation at the current temperature. The dependence of pressure on temperature may be expressed by

$$p_{(\mathrm{l})}=p_{(\mathrm{l})}(T)=p_{(l)^*}\exp(-T_*/T), \quad (9)$$

where $p_{(l)^*}$ and T_* are the empirical approximate parameters.

In addition, assume that the pressure in a gas phase satisfies the law of Dalton

$$p=p_{(\mathrm{l})}+p_{(\mathrm{g})}, \quad (10)$$

$$p_{(\mathrm{l})}=\rho^0_{\mathrm{g}(\mathrm{l})}R_{(\mathrm{l})}T,\quad p_{(\mathrm{g})}=\rho^0_{\mathrm{g}(\mathrm{g})}R_{(\mathrm{g})}T,$$

$$\rho^0_{\mathrm{g}(\mathrm{l})}+\rho^0_{\mathrm{g}(\mathrm{g})}=\rho^0_\mathrm{g},\quad R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})}\right)+R_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})}=R_\mathrm{g},$$

where $\rho^0_{\mathrm{g}(\mathrm{l})}$, $\rho^0_{\mathrm{g}(\mathrm{g})}$ and $R_{(\mathrm{l})}$, $R_{(\mathrm{g})}$ are the true densities and gas constants according to average and light components in a gas phase.

On the basis of Eqs. (8)–(10), excepting $p_{(\mathrm{l})}$ and $p_{(\mathrm{g})}$, we get

$$p=p_{(\mathrm{l})}(T)+G_{(\mathrm{g})}k_{\mathrm{l}(\mathrm{g})}, \quad (11)$$

$$\frac{p_{(\mathrm{l})}(T)}{p}=\frac{R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})}\right)}{R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})}\right)+R_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})}}=B. \quad (12)$$

Differentiating (11) and (12) with respect to z, we have

$$\frac{\mathrm{d}p}{\mathrm{d}z}=\frac{\mathrm{d}p_{(\mathrm{l})}(T)}{\mathrm{d}z}+G_{(\mathrm{g})}\frac{\mathrm{d}k_{\mathrm{l}(\mathrm{g})}}{\mathrm{d}z},$$

$$\frac{\mathrm{d}p_{(\mathrm{l})}(T)}{\mathrm{d}z}=p'_{(\mathrm{l})}(T)\frac{\mathrm{d}T}{\mathrm{d}z}=B\frac{\mathrm{d}p}{\mathrm{d}z}+B'p\frac{\mathrm{d}k_{\mathrm{g}(\mathrm{g})}}{\mathrm{d}z},$$

$$p'_{(\mathrm{l})}(T)=\frac{\mathrm{d}p_{(\mathrm{l})}(T)}{\mathrm{d}T}=\frac{T_*p_{(\mathrm{l})}(T)}{T^2},$$

$$B'=\frac{\mathrm{d}B}{\mathrm{d}k_{\mathrm{g}(\mathrm{g})}}=-\frac{R_{(l)R_{(\mathrm{g})}}}{\left[R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})}\right)+R_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})}\right]^2}.$$

Consequently,

$$\frac{\mathrm{d}k_{\mathrm{l}(\mathrm{g})}}{\mathrm{d}z}=\left(\frac{\mathrm{d}p}{\mathrm{d}z}-p'_{(\mathrm{l})}(T)\frac{\mathrm{d}T}{\mathrm{d}z}\right)\Big/G_{(\mathrm{g})}, \quad (13)$$

$$\frac{\mathrm{d}k_{\mathrm{g}(\mathrm{g})}}{\mathrm{d}z}=\left(p'_{(\mathrm{l})}(T)\frac{\mathrm{d}T}{\mathrm{d}z}-B\frac{\mathrm{d}p}{\mathrm{d}z}\right)\Big/B'p. \quad (14)$$

Eqs. (4), (7), (13) and (14) form a system of ordinary differential equations, the solution of which yields the pressure profile, temperature, and mass concentration of components along the well.

It is necessary to set boundary conditions on the bottom hole and head of the well for solving the system. We assume that the output of well m is defined by reservoir pressure p_b and bottom hole pressure p_f. Thus,

$$m=K(p_\mathrm{b}-p_\mathrm{f})\quad\text{or}\quad m=K'(p_\mathrm{b}^2-p_\mathrm{f}^2).$$

The indicated equations correspond to two extreme cases: first—when a product of a well is a liquid, second—a gas.

Let us consider parameters K and K' as constants. We shall consider the temperature on the bottom hole, T_f, equal to the temperature of a reservoir. Concentration for a light component in liquid and gas phases $(k_{\mathrm{l}(\mathrm{g})\mathrm{f}},k_{\mathrm{g}(\mathrm{g})\mathrm{f}})$ on a bottom hole of a well we shall define through bottom hole pressure and temperature from (11) and (12):

$$k_{\mathrm{l}(\mathrm{g})\mathrm{f}}=\frac{p_\mathrm{f}-p_{(\mathrm{l})}(T)}{G_{(\mathrm{g})}}, \quad (15)$$

$$\frac{p_{(\mathrm{l})}(T_\mathrm{f})}{p_\mathrm{f}}=\frac{R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})\mathrm{f}}\right)}{R_{(\mathrm{l})}\left(1-k_{\mathrm{g}(\mathrm{g})\mathrm{f}}\right)+R_{(\mathrm{g})}k_{\mathrm{g}(\mathrm{g})\mathrm{f}}}. \quad (16)$$

To find from (15) and (16) mass concentration $k_{\mathrm{l}(\mathrm{g})\mathrm{f}}$ and $k_{\mathrm{g}(\mathrm{g})\mathrm{f}}$, we shall define by numerical integration of a system of Eqs. (4), (7), (13) and (14) the distribution of hydrodynamic parameters on a well and on a head.

For closing the system of Eqs. (4), (7), (13) and (14), it is necessary to set the intensity of growth of deposits of paraffin on the internal walls of an elevating column of a well, as well as the intensity of heat transfer, Q_w, in the surrounding rock.

3. Heat exchange of a well with surrounding rock

The well cannot be considered as a thermodynamically isolated system. For complete research of the heat transfer of a two-phase flow in an elevating column, consideration of processes of thermal interaction of well flows with the surrounding medium is required. It is necessary to note that temperature fields in a system "well-surrounding rock" are non-stationary in the general case. However, some estimates show that at characteristic rates of change of temperature fields for well flows, the influence of their non-stationarity on a factor of heat transfer from a flow to a wall of an elevating column is not substantial. Therefore, the non-stationary process of a heat exchange can be simulated by the usual methods of stationary heat transfer.

To determine the intensity of a heat exchange between a flow and wall of a well (or solid phase on a wall of a well), let us allocate two sites on a depth of a well. On the first site, located between the bottom hole and the section of well where temperature on the internal wall T_o reaches temperature of crystallization for the heavy component T_e, the deposits of a solid phase takes place. For heat flux we can write

$$q_w = \beta_w(T - T_o),$$

where β_w is the heat transfer coefficient, depending on the structure of gas–liquid flow in the well, and on features of flow in layers adjacent to the solid phase or walls of a channel.

For definition of temperature of an internal wall of a well it is necessary to consider the problem of an external heat exchange; and if the site of the section where T_o reaches the temperature of crystallization for the heavy component T_e is above a base of permanent frost, it is important to take into account the formation of melting zones.

The second site is between the section of beginning of formation of a solid phase and the head of a well. The heat flux on this site of a well, where paraffin deposits occur, is

$$q_w = \beta_w(T - T_\sigma), \tag{17}$$

where T_σ is the temperature of an internal surface of a solid phase.

By consideration of an unsteady mode of heat transfer from an internal wall of a channel in layers of ground [2] through which the well is drilled, usually heat transfer is described with the help of a factor

$$\beta = \frac{1}{R_N \sum_{i=1}^{N} \lambda_i^{-1} \ln(R_i/R_{i-1})} \quad [R_N = R_c].$$

Here R_i is the external radius of the ith layer, λ_i the thermal conductivity in ith layer, R_c the external radius of the well; thus $i = 1$ corresponds to an elevating column, $i = N$ the layer, contiguous to surrounding rock.

The account of natural convection of a medium in the annular space can be made with the help of a multiplier for thermal conductivity of a medium in a specified volume

$$Nu = 0.049(Gr\,Pr)^{1/3} Pr^{0.074},$$

where Nu, Gr, and Pr are numbers of Nusselt, Grashof, and Prandtl.

For valuation of influence of various types of isolation numerical accounts are conducted. Thus the following values are used: $H = 3000$ m, $p_b = 22.7$ MPa, $p_f|_{t=0} = 13.6$ MPa, $T_f = 353$ K, $R_0 = 0.0315$ m, $R_c = 0.125$ m, $m_l = 0.463$ kg/s, $m_g = 0.083$ kg/s, $\rho_l^o = 850$ kg/m^3, $\mu_l = 4.1 \times 10^{-3}$ kg/m s, $\mu_g = 1.25 \times 10^{-5}$ kg/m s, $R_{(l)} = 52$ m^2/s^2 K, $R_{(g)} = 520$ m^2/s^2 K, $\lambda_l = 0.13$ kg m/s^3 K, $c_l = 2100$ m^2/s^3 K, $c_g = 2500$ m^2/s^3 K, $t_l^* \cong 2R/v_l$, $l_{(g)}^{lg} = 10^4$ m^2/s^2 and $\varepsilon = 1.5 \times 10^{-5}$ m. Here H-depth of a well. We shall assume that the temperature on the external border of a well is equal to the geothermal $T_0^{(1)}$. Here, it is necessary to note that this is the limiting case, when the heaviest heat outflow from a well is realized. In the general case, warming of the ground around the well is taken into account.

It is apparent from Fig. 2 that filling the annular space of a well with a substance with a smaller thermal conductivity largely improves the temperature mode in the bore of the well. If a gas in the annular space is in a state of natural convection, it results in an increase of thermal losses of the well. Using heat-insulation pipes

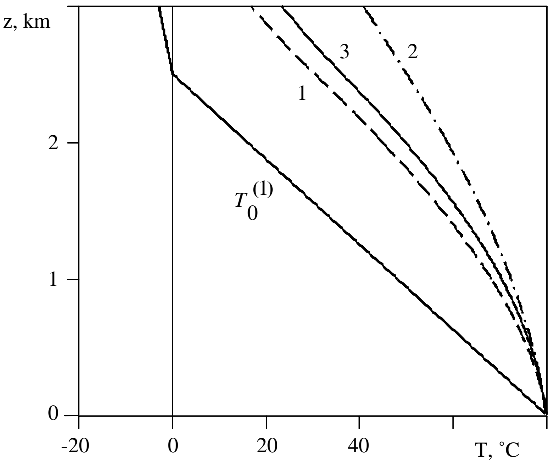

Fig. 2. The distribution of the average temperature of a gas–liquid flow on a well depending on the type and condition of substance in the annular space. Curves 1, 2 and 3 correspond to cases when the annular space is filled: by a liquid (petroleum), gas, and gas in a natural convection condition, respectively.

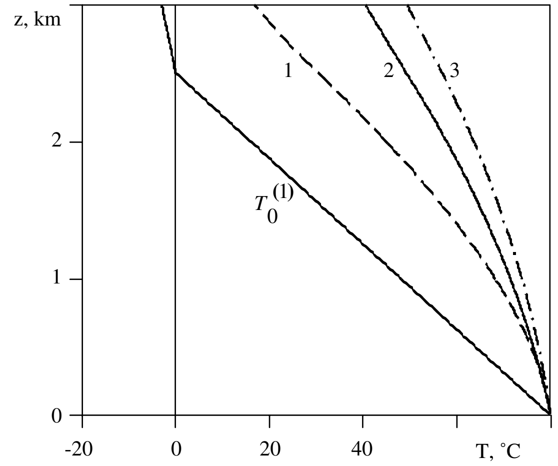

Fig. 3. The distribution of the average temperature of a gas–liquid flow in a well depending on the thickness dh of heat isolating material (polyurethane foam, $\lambda = 0.0067$ W/m K) on the external border of an elevating column. Curves 1, 2, 3 correspond to d$h = 0$, 5, 10 mm, respectively; the remaining volume of the annular space is filled by petroleum.

(Fig. 3) permits improvement of temperature conditions in the elevating column of the well. However, manufacturing the equipment is difficult and is not always economically justified.

The temperature conditions in an elevating column of a well largely depend on the condition of the surrounding annular space. Except for heat exchange with the surrounding rock, the thermal mode of operations of a well is determined by the additional influence of two main factors: by the velocity of flow and by the adiabatic expansion of a two-phase mix. Reduction of well head pressure decreases pressure on the bottom hole and increases output of a well. But with reduction of heat outflow, temperature losses of a gas–liquid flow, connected with adiabatic expansion, grow. Therefore, at some particular well head pressure temperature in a bore of a well is highest (Fig. 4).

As mentioned above, for the specification of temperature of the internal wall of the channel it is necessary to consider the interaction of a well with the surrounding rock. For operation of a well in conditions of permanent frost, the interaction of a well with the surrounding rock at certain depths is accompanied by a melting of the frozen ground near the well. Therefore, in the general case, it is necessary to include thermal conductivity in phase transitions for the description of the external heat exchange of the well.

Heat transfer between a well and surrounding rock in a zone, located below the base of permanent frost, or in a zone of permanent frost up to the moment of occurrence of melting rock $t^{(1)}$ is described by the heat equation.

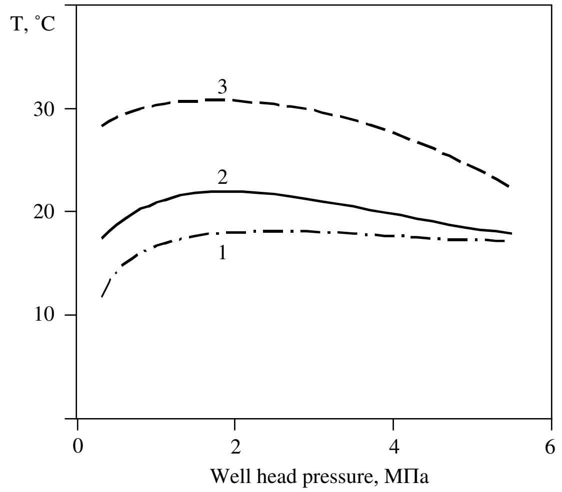

Fig. 4. The dependence of the average temperature of a gas–liquid flow in an elevating column on well head pressure at various depths. Curves 1, 2 and 3 correspond to $h = 0$, 200 and 500 m, respectively.

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1} \frac{\partial}{\partial r}\left(r \frac{\partial T^{(1)}}{\partial r} \right), \quad 0 < t < t^{(1)}, \ r > R_c, \tag{18}$$

$$\chi^{(i)} = \lambda^{(i)} / \rho^{(i)} c^{(i)}.$$

Here and below $\rho^{(i)}$, $c^{(i)}$, $T^{(i)}$, $\lambda^{(i)}$, $\chi^{(i)}$ are density, specific heat, temperature, thermal conductivity and thermal diffusivity coefficients. The superscript in brackets ($i = 1$) corresponds to parameters of the surrounding rock in the zone, of permanent frost located below the base, or parameters of frozen rock in a zone of permanent frost; ($i = 2$) corresponds to parameters of melting rock.

On a border of contact of a well with a rock we shall write the condition

$$-\lambda^{(1)} \frac{\partial T^{(1)}}{\partial r} = \beta (T_0 - T^{(1)}), \quad 0 < t < t^{(1)}, \ r = R_c. \tag{19}$$

At infinity, we shall require a condition of limitation of temperature

$$\partial T^{(1)} / \partial r = 0, \quad 0 < t < t^{(1)}, \ r = \infty.$$

In addition, we shall write a condition of equality of heat flows through the external and internal walls of a well:

$$\lambda^{(1)} R_c \left(\frac{\partial T^{(1)}}{\partial r} \right)_{R_c} = \lambda_s R_0 \left(\frac{\partial T_s}{\partial r} \right)_{R_0}. \tag{20}$$

The described external thermal problem for these considered processes is effectively solved on the basis of an integrated method [4], according to which the distribution of temperature around the well is stipulated as

$$T^{(1)} = C_1 \ln(r/R_c) + C_2(r/R_c) + C_3, \tag{21}$$

with boundary conditions

$$T^{(1)} = T_0^{(1)}, \quad \partial T^{(1)}/\partial r = 0, \quad r = R_*(t). \tag{22}$$

Parameters C_1, C_2 and C_3 can be found by solving function (21) subject to boundary conditions (19) and (22).

The radius of thermal influence of the well $R_*(t)$ is determined on the basis of (18). Multiplying this equation by r and integrating from a surface of the well ($r = R_c$) up to the border of influence ($r = R_*$), we have

$$\int_{R_c}^{R_*} r\frac{\partial T^{(1)}}{\partial t}\,\mathrm{d}r = \chi^{(1)} \int_{R_c}^{R_*} \frac{\partial}{\partial r}\left(r\frac{\partial T^{(1)}}{\partial r}\right)\mathrm{d}r.$$

Substituting for $T^{(1)}$ the expression from (21), and taking into account conditions (22), we have an ordinary differential equation for determination of the radius of thermal influence of a well

$$\begin{aligned}\frac{\mathrm{d}\widetilde{R}_*}{\mathrm{d}t} = {} & 12(\chi^{(1)}/R_c^2)(1-\widetilde{R}_*)\Big(\widetilde{R}_* - 1 + \beta^{(1)}\Big[\widetilde{R}_*\ln(\widetilde{R}_*) \\ & - \widetilde{R}_* + 1\Big]\Big)\Big/\Big\{3[\beta^{(1)} - 7/3] + 3\widetilde{R}_*[2-\beta^{(1)}] \\ & + 3\widetilde{R}_*^2[1-\beta^{(1)}] + 2[\beta^{(1)} - 3]\ln(\widetilde{R}_*) \\ & - 2\beta^{(1)}\widetilde{R}_*^3\ln(\widetilde{R}_*) + \widetilde{R}_*^3[3\beta^{(1)} - 2]\Big\},\end{aligned} \tag{23}$$

$\widetilde{R}_* = R_*/R_c, \quad \beta^{(i)} = \beta R_c/\lambda^{(i)} \ (i = 1,2).$

The above equations determine the thermal fields around a well until the moment the temperature of the rock on a surface of the well reaches the melting point of frozen rock $T^{(1,2)}$. For the description of further process of a heat exchange between a well and rock it is necessary to take into account melting zones between the well and the surface of a melting of frozen rock with radius $R^{(1,2)}$. Pursuant to these external thermal problems around the well we shall write

$$\frac{\partial T^{(2)}}{\partial t} = \chi^{(2)} r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(2)}}{\partial r}\right), \quad t > t^{(1)}\ R_c < r < R^{(1,2)}, \tag{24}$$

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(1)}}{\partial r}\right), \quad t > t^{(1)},\ R^{(1,2)} < r < \infty, \tag{25}$$

$$-\lambda^{(2)}\frac{\partial T^{(2)}}{\partial r} = \beta(T_0 - T^{(2)}), \quad t > t^{(1)},\ r = R_c, \tag{26}$$

$$T^{(1)} = T^{(2)} = T^{(1,2)}, \quad r = R^{(1,2)}, \tag{27}$$

$$-\lambda^{(2)}\frac{\partial T^{(2)}}{\partial r} + \lambda^{(1)}\frac{\partial T^{(1)}}{\partial r} = \rho^{(1)} l^{(1)}\frac{\mathrm{d}R^{(1,2)}}{\mathrm{d}t} \quad r = R^{(1,2)}, \tag{28}$$

$$\lambda^{(2)} R_c\left(\frac{\partial T^{(2)}}{\partial r}\right)_{R_c} = \lambda_s R_0\left(\frac{\partial T_s}{\partial r}\right)_{R_0}, \tag{29}$$

$$\partial T^{(1)}/\partial r = 0, \quad r = \infty,$$

where $l^{(1)}$ is the melting point of frozen rock.

Similarly, for $T^{(1)}$ the following ordinary differential equation results from (22), (25) and (27)

$$\begin{aligned} & -\frac{\mathrm{d}\widetilde{R}^{(1,2)}}{\mathrm{d}t}\left(\frac{\widetilde{R}^{(1,2)}}{2\widetilde{R}_*}\ln\frac{\widetilde{R}_*}{\widetilde{R}^{(1,2)}} - \frac{\widetilde{R}^{(1,2)}}{4\widetilde{R}_*} + \frac{\widetilde{R}^{(1,2)^2}}{3\widetilde{R}_*^2} - \frac{\widetilde{R}_*}{12\widetilde{R}^{(1,2)}}\right) \\ & \quad + \frac{\mathrm{d}\widetilde{R}_*}{\mathrm{d}t}\left(\frac{\widetilde{R}^{(1,2)^2} + \widetilde{R}_*^2 + \widetilde{R}_*\widetilde{R}^{(1,2)}}{6\widetilde{R}_*^2}\ln\frac{\widetilde{R}_*}{\widetilde{R}^{(1,2)}} - \frac{1}{4} + \frac{\widetilde{R}^{(1,2)^2}}{4\widetilde{R}_*^2}\right) \\ & \quad = \frac{\chi^{(1)}}{R_c^2\widetilde{R}_*}\left(\ln\frac{\widetilde{R}_*}{\widetilde{R}^{(1,2)}} + \frac{\widetilde{R}^{(1,2)}}{\widetilde{R}_*} - 1\right),\end{aligned} \tag{30}$$

$\widetilde{R}^{(1,2)} = R^{(1,2)}/R_c.$

Temperature structures in a melting zone are defined on the basis of the method of consecutive shift of stationary states [6], according to which the distribution of temperature satisfies the equation

$$r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(2)}}{\partial r}\right) = 0, \quad R_c < r < R^{(1,2)}.$$

Incorporating boundary conditions (26) and (27), we have

$$T^{(2)} = T^{(1,2)} + \frac{\beta^{(2)}(T_0 - T^{(1,2)})\ln\left(\widetilde{R}^{(1,2)}R_c r^{-1}\right)}{1 + \beta^{(2)}\ln\left(\widetilde{R}^{(1,2)}\right)}. \tag{31}$$

On the basis (21) and (31) we have

$$\begin{aligned}\left(\frac{\partial T^{(1)}}{\partial r}\right)_{R^{(1,2)}} &= \frac{T_0^{(1)} - T^{(1,2)}}{R_c} \\ & \quad \times \frac{\widetilde{R}_*/\widetilde{R}^{(1,2)} - 1}{\widetilde{R}_*\ln\left(\widetilde{R}_*/\widetilde{R}^{(1,2)}\right) + \widetilde{R}^{(1,2)} - \widetilde{R}_*}, \\ \left(\frac{\partial T^{(2)}}{\partial r}\right)_{R^{(1,2)}} &= \frac{T^{(1,2)} - T_0}{R_c}\frac{\beta^{(2)}}{\widetilde{R}^{(1,2)}\left\{\beta^{(2)}\ln\left(\widetilde{R}^{(1,2)}\right) + 1\right\}}.\end{aligned}$$

Substituting the indicated expressions in the condition (28) we obtain the following ordinary differential equation

$$\begin{aligned}\frac{\mathrm{d}\widetilde{R}^{(1,2)}}{\mathrm{d}t} = {} & \frac{\lambda^{(2)}\left(T_0 - T^{(1,2)}\right)}{R_c^2\rho^{(1)}l^{(1)}}\frac{\beta^{(2)}}{\widetilde{R}^{(1,2)}\left\{\beta^{(2)}\ln\left(\widetilde{R}^{(1,2)}\right) + 1\right\}} \\ & + \frac{\lambda^{(1)}\left(T_0^{(1)} - T^{(1,2)}\right)}{R_c^2\rho^{(1)}l^{(1)}}\frac{\widetilde{R}_*/\widetilde{R}^{(1,2)} - 1}{\widetilde{R}_*\ln\left(\widetilde{R}_*/\widetilde{R}^{(1,2)}\right) + \widetilde{R}^{(1,2)} - \widetilde{R}_*}.\end{aligned} \tag{32}$$

Eqs. (30) and (32) will form a system for determining $R_*(t)$ and $R^{(1,2)}(t)$.

4. Kinetics of paraffin deposits

The operation of gas-oil and gas condensate wells is usually complicated by the occurrence of two types of deposits. The first type of deposit is gas hydrate. A necessary condition for the formation of such deposits is the availability of water (or its vapor) and hydrocarbon components, which when dissolved in water at certain temperatures and pressure will form a solid phase [8]. The second type of deposit, which is usually observed during operation of gas-oil wells, represents high-molecular hydrocarbon systems, which we call paraffin deposits. For formation of paraffin deposits, the two-phase mix must contain heavy (high-molecular) components (the direct precursors of deposits). In addition, the temperature in a well (especially near its walls) should allow the existence of a solid phase. In particular, the temperature should becomes less or equal to the temperature of crystallization of the heavy components.

The literature shows that the most probable mechanism of formation of paraffin deposits is crystallization. Diffusion and thermodiffusion carry the heavy hydrocarbon component to the well wall resulting in formation and growth of the solid phase on this wall. The intensity of paraffin deposits depends on the flow velocity or output of the well. Therefore, in general, it is necessary to use the following expression for an evaluation of the boundary "gas–liquid flow—paraffin"

$$\rho_s^0 \frac{\partial \delta_s}{\partial t} = -j_s - j_s'(w) \quad (R = R_0 - \delta_s).$$

Here j_s is the intensity of formation of paraffin connected with mass transfer of heavy components to the surface of the solid phase and heat exchange of the well with the surrounding rock; $j_s'(w)$ is the intensity of ablation of paraffin deposits by gas–liquid flow. We will assume

$$j_s'(w) = \gamma w |w|,$$

where γ-factor of proportionality, which must be calculated experimentally. Such data are available in the literature.

We assume the formation of a solid phase (paraffin) is basically defined by loss of fluidity of a liquid near the walls because of its stagnation with decreasing temperature ($j_s'(w) = 0$). This assumption means that the liquid contains a surplus of heavy components, inducing the solid phase, and the intensity of the process of depositing a solid phase is defined by the condition of the thermal balance on a surface of deposits

$$l^{ls} j_s = -q_w - \lambda_s \left(\frac{\partial T_s}{\partial r} \right)_R, \quad \frac{\partial \delta_s}{\partial t} = \frac{j_s}{\rho_s^0}. \tag{33}$$

We shall assume that the internal surface of a solid phase is an isothermal surface, the temperature of which, T_σ, is equal to the temperature of crystallization, T_e, for a hydrocarbon mix flowing in the well.

As the increase of a layer of paraffin on a wall of a well occurs reasonably slowly (the characteristic time of a complete closing of a carrying section of a well is considerably more than the characteristic time of establishment of stationary structure temperatures inside the solid phase $t_s = \delta_s^2/\chi_s$, χ_s is the thermal diffusivity of a paraffin), we shall suggest that the distribution of temperature in the firm layer T_s for each moment of time satisfies the equation

$$r^{-1} \frac{\partial}{\partial r} \left(\chi_s r \frac{\partial T_s}{\partial r} \right) = 0 \quad (R < r < R_0). \tag{34}$$

On the basis of (34) we shall state the following expression for the distribution of temperature in a firm layer

$$T_s = T_0 + \frac{T_\sigma - T_0}{\ln(R/R_0)} \ln(r/R_0). \tag{35}$$

In view of expressions (17) and (35), Eq. (33) can be transformed to

$$\rho_s^0 l^{ls} \frac{\partial \delta_s}{\partial t} = -\beta_w (T - T_\sigma) + \beta_s (T_0 - T_\sigma), \tag{36}$$

$$\beta_s = \frac{\lambda_s}{R \ln(R/R_0)}.$$

The above description of the process of deposit of a firm phase assumes that its intensity is completely defined by the condition of the thermal balance. In the more general case, the intensity of deposit depends on a mass transfer of a heavy component to a surface of a solid phase. Therefore, the intensity of formation of a solid phase can be described by

$$j_s = \frac{\rho_l^0 D Sh}{2R} \left(k_{l(s)} - k_{l(s)e} \right),$$

$$Sh = \sqrt{2Rv_l/D},$$

where D is the coefficient of diffusion; $k_{l(s)}$ and $k_{l(s)e}$ are the mass average concentration of a heavy component in a liquid and equilibrium concentration, respectively, of a heavy component at a temperature equal to the temperature of a surface of the solid phase. The Sherwood number, Sh, here assumes that the mass transfer occurs in a thin diffusion boundary layer with thickness $\delta^{(D)}$ ($\delta^{(D)} \ll R$) near to a border "solid phase-liquid". The characteristic thickness of this layer is

$$\delta^{(D)} \cong \sqrt{D \cdot t_l^*},$$

where t_l^* is the some characteristic time of contact of a particle of a liquid with a wall of a channel and which, in turn, can be given by $t_l^* \cong 2R/v_l$.

On the basis of the Schreder equation relating $k_{l(s)e}$ and T_σ, it is possible to write

$$k_{l(s)e} = k^*_{l(s)e}\exp(-T^*_s/T_\sigma),$$

where $k^*_{l(s)e}$ and T^*_s are the empirical approximation parameters.

We assume that the dependence of $k_{l(s)e}$ on T_σ is linear. Therefore, the intensity of formation of a solid phase will be

$$\rho_s^0\frac{\partial\delta_s}{\partial t} = K_m(T_e - T_\sigma), \qquad (37)$$

$$K_m = \frac{\rho_l^0 D Sh}{2R}\left(\frac{\partial k_{l(s)e}}{\partial T_\sigma}\right)_{T_e}.$$

Eliminating T_σ from (36) and (37), we may write

$$\frac{\partial\delta_s}{\partial t} = \frac{-\beta_w(T - T_e) + \beta_s(T_0 - T_e)}{\rho_s^0[l^{ls} + (\beta_w - \beta_s)/K_m]}.$$

The temperature of a surface of a solid phase can be determined from the expression

$$T_\sigma = \frac{\beta_w T + K_m l^{ls} T_e - \beta_s T_0}{\beta_w + K_m l^{ls} - \beta_s}.$$

On the basis of (20), (21) and (35), it is possible to develop an expression for temperature on the internal surface of a wall of a well depending on the thickness of deposits and the radius of influence of a well in a zone below the permanent frost line (Fig. 5)

$$T_0 = T_\sigma - \frac{B_1\left(T_0^{(1)} - T_\sigma\right)\ln(R/R_0)}{\lambda_s/\lambda^{(1)} - B_1\ln(R/R_0)},$$

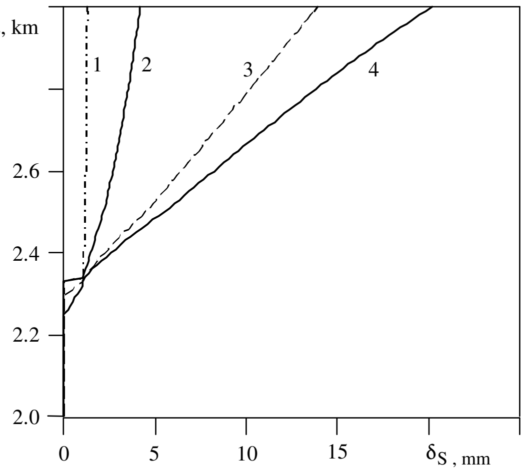

Fig. 5. The profile diagram of deposits of paraffin on internal walls of an elevating column in various moments of time. Curves 1, 2, 3 and 4 correspond to $t = 1$, 4, 7 and 8 days, respectively.

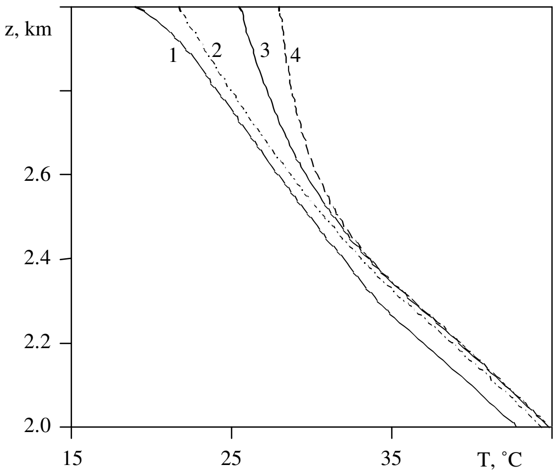

Fig. 6. The distribution of the average temperature of a gas–liquid flow on the top site of a well depending on thickness of deposit of paraffin, the structure of which is shown in Fig. 5.

$$B_1 = \frac{\beta^{(1)}\left(\widetilde{R}_* - 1\right)}{\widetilde{R}_* - 1 + \beta^{(1)}\left(\widetilde{R}_*\ln(\widetilde{R}_*) - \widetilde{R}_* + 1\right)}.$$

In a zone of permafrost, the expression for temperature on the internal surface of a channel of a well can be written, using (29), (31) and (35), as follows

$$T_0 = T_\sigma + \frac{\beta^{(2)}\left(T^{(1,2)} - T_\sigma\right)\ln(R/R_0)}{\beta^{(2)}\ln(R/R_0) - \lambda_s\left\{1 + \beta^{(2)}\ln(\widetilde{R}^{(1,2)})\right\}/\lambda^{(2)}}.$$

Numerical calculations conducted on the kinetics of paraffin deposits yielded the following parameters: $T_e = 303$ K, $\rho_s^0 = 900$ kg/m^3, $\lambda_s = 0.27$ kg m/s^3 K, $l^{ls} = 2\times10^5$ m^2/s^2, where ρ_s^0, λ_s and l^{ls} are the true density, thermal conductivity and heat of crystallization of paraffin (Fig. 6).

Obviously, there are other competitive factors, namely: the pressure is increased with growth of deposit and it results, at constant well head pressure and temperature in a bottom hole of a well, in a decrease of temperatures. In addition, the process of paraffin deposit leads to the reduction of the cross-section of a channel and accordingly to a decrease of output of a well. The more slowly a gas–liquid flow moves, the larger the quantity of heat passed to the surrounding mountain rock for a given time of rise to head.

5. Conclusion

The main results of this work may be stated as follows.

(1) A mathematical model of gas-oil well exploration is developed, within the framework of which such complex interconnected circumstances as hydro and heat dynamics of a gas–liquid flow in a vertical channel with variable section, deposits of paraffin on internal walls of elevating column of a well, and heat exchange of a well with surrounding rock with formation of melting zones are discussed.

(2) Within the framework of the constructed model, numerical calculations establish that non-monotonous dependence of a temperature mode in an elevating column from well head pressure of a well exists.

(3) Paraffin deposits not only do not render an appreciable heat-isolating effect on rates of growth, but on the contrary make this process more intense with the decrease of temperatures in the elevating column due to increases of pressure on a well and reduction of cross-sectional areas.

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